Kim H. Veltman

Developments in Perspective

Published in: *Visual Mathematics*, ed. Michele Emmer, Cambridge, Mass.: M.I.T. Press, 1993, pp. 199-205.

Abstract

A growing historical awareness means that there have been many more books on Renaissance perspective published in the twentieth than in the fifteenth and sixteenth centuries combined. Renaissance perspective focussed on geometrical space. Twentieth century versions of perspective focus on visual space and hence entail a variety of alternative picture planes: spherical, cylindrical and even polygonal. New goals of art have influenced these developments as have computer algorithms, cartographical methods, new awareness of illusions, the development of virtual reality and fractals.

1. Introduction

Arnason, in his standard *History of Modern Art* described Renaissance art as "imitations of nature" and claimed that: "Perhaps the greatest revolution of early modern art lay in the abandonment of this attitude and the perspective technique that made it possible". [1]Such claims have led to a widespread view that interest in perspective died in the twentieth century. Even a quick glance at the number of books published on perspective since the fifteenth century[1] reveals that this is not the case (fig. 1).

1400-1499	1
1500-1599	456
1600-1699	732
1700-1799	849
1800-1899	2714
1900-1989	2801

Fig. 1. Books on perspective printed since 1400.

[1]. This list is from the author's *Sources of perspective* which, along with *Literature of perspective* and two volumes of bibliography is to be published by the Saur Verlag.

This partly reflects a growing historical awareness. There have been more editions of fifteenth and sixteenth century authors on perspective in our century than ever before, including first critical editions of Alberti, Filarete, Francesco di Giorgio and now Piero della Francesca. [2]Piero della Francesca is all the more interesting because his *Flagellation* is one of the very few Renaissance paintings which has a carefully determined perspectival construction. Ever since Wittkower and Carter's classic article[3] there has been a fascination among scholars with finding new ways to reconstruct the perspectival lines in this painting as witnessed by the first contribution in this section by Geatti and Fortunati which uses computer graphics. In Renaissance treatises on perspective, certain themes such as the regular solids were particularly popular. The second and third contributions in this section by Emmer and Saffaro can be seen as a direct continuity of that tradition. The third article by Koch and Tarnai as well as the fourth contribution Koptsik can be seen as poetic variations on this theme of symmetry and regularity.

Antiquarian and nostalgic interest comprise but one aspect of modern concern for perspective. Partly because there are now a number of different goals of art, artists are exploring a whole range of alternative methods. Some of these experiments relate perspective to what might be termed a personalized optics. Others reflect a new awareness of transformations brought about by mapping, computers and virtual reality. Meanwhile the advent of fractals has raised but not answered new philosophical problems concerning the parameters within which perspective is valid. It will be useful to review briefly the context for and characteristics of Renaissance perspective before considering each of these topics in turn.

2. Renaissance Perspective

In Antiquity, Euclid's *Optics* focussed on how things appear to the eye: a concern which is now termed psychological optics. Euclid's treatise also contained four propositions on surveying. This introduced a quantitative dimension into an otherwise qualitative discussion. In the course of the Middle Ages, particularly thanks to thinkers such as Ibn al-Haytham (Alhazen) in the Arabic tradition, this concern with potentially quantitative, objective aspects of sight gained greatly in importance.[4] By the 1270's, when Witelo wrote his great compendium on optics, (which was termed *perspectiva* in Latin), he was concerned not just with the eye but also with instruments "for the certification of sight" such as quadrants and astrolabes. A nexus thus evolved which linked optics, surveying, astronomy and instrumentation. This concern with certification of sight led to practical demonstrations which involved representation. Brunelleschi's demonstration was in this tradition. In the fifteenth and sixteenth centuries this became known as practical (linear) perspective, for which Euclidean optics was assumed to provide the theory.

3. Vision and Geometry

Basic to the Euclidean theory of vision was an angle axiom which denied a simple inverse relation between (apparent) size and distance. Linear perspective introduced an inverse size-distance axiom. Panofsky assumed that theories of vision and representation were necessarily linked and hence believed that the advent of linear perspective required a change in the theory of vision.[5] He cited Pena's 1557 edition of Euclid's *Optics* as evidence. In fact the Euclidean theory of vision remained unaffected by perspective. Perspective, which had been linked with optics, became linked increasingly with geometry. In the sixteenth century, this occured particularly in Urbino (Commandino, Benedetti, Guidobaldo del Monte).

In the seventeenth century Paris became the world centre for mathematical perspective (e.g. Aleaume, Desargues, Niceron, Bosse). Thus Abraham Bosse, the first professor of perspective of the French Academy, could urge painters that they must draw what is there (Euclidean geometry) and not what is seen (Euclidean theory of vision).[6] His colleagues, particularly Charles Le Brun, preferred to expel Bosse from the Academy rather than face the consequences of this clear statement of logic. From the 1660's to the 1820's artists either a) chose limited conditions where the effects of vision and perspective co-incided or b) spoke in general terms about (Euclidean) vision and (linear) perspective. The rise of descriptive geometry which claimed to offer universal principles for representation led to the first serious claims that representation and vision must be co-incident. Hence Panofsky was reading back into the sixteenth century a development that occured in the early nineteenth century.

The nineteenth century also brought other developments. Mathematicians explored various alternatives to (rectilinear) Euclidean space. These experiments eventually had consequences for modern art as Henderson and Corrada show in the fifth and sixth contributions to this section. Meanwhile, physiologists became aware of serious problems with earlier analogies between the eye and camera obscura (Leonardo, Kepler, Scheiner). Helmholtz discovered that curved lines, when seen from nearby, appear straight and hence suggested a distinction between physical space which was Euclidean, and visual space which was non-Euclidean (and possibly Riemannian). Mach explored this distinction in his *Analysis of Sensations* and devoted a chapter to it in his *Knowledge and Error* (1905). The emergent schools of psychology focussed on different aspects of this distinction. The Berlin school, later the Gestalt school, emphasized geometrical space. The Leipzig school (Wundt, Titchener) emphasized visual space. In the twentieth century this distinction has remained with physiologists such as Doesschate. Sometimes the names for the elements have changed. Sir Ernst Gombrich, for example, referred to visual space as the optical world or the mirror and to geometrical space as the physical world, the experienced world or the map.[7]

The late J. J. Gibson called visual space the visual field and linked geometrical space with the visual world.[8] There is more to this change of terms than is at first apparent. In the nineteenth century it was generally assumed that visual space was subjective and geometrical space was objective. In Gibson's approach both are susceptible to measurement and hence in some sense objective. In Gibson's formulation there is also no opposition between vision and geometry. Geometry applies to both physical space and to (psychological) visual space. The question remains whether the same branch of geometry applies to both kinds of space? Those who claim that (linear) perspective is dead often mean that artists have given up trying to record physical space and are focussing on visual space with its non-Euclidean surfaces. There is something to this but it is not the whole story.

4. Multiple Goals

The history of art is often told as if art had only one goal. Arnason's *History of modern art* can easily be read as if this were the case: for a long time the goal was imitation of nature and hence perspective was important. Then artists discovered the challenge of abstraction as a goal and abandoned perspective. One of the enduring contributions of Sir Ernst Gombrich has been to emphasize multiple functions or goals of art: e.g. magic (which used to be termed primitive art); pattern (*Sense of Order*), mimesis (*Art and Illusion*, *Image and the Eye*, *Illusion in Nature and Art*), expression and abstraction (*Meditations on a Hobbyhorse*) and symbolism (*Symbolic Images*). In this approach some functions of art do not need and sometimes even preclude perspective, while others encourage its use.

The twentieth century has added new functions or goals. One might might be termed exploring, which can be subdivided into the mental world, the perceptual world and chance. The third of these, of which Jackson Pollock is an excellent example, aims to remove any clear one-to-one correspondence between artist and art. This goal, which results in abstract painting, was for a time frequently identified with modern art. It is becoming ever more obvious, however, that the other two areas of exploring, namely, the mental and the perceptual world, have inspired a much richer repertoire of images. Exploration of the mental world has led to depiction of dreams, phantasies and other psychological dimensions. As a result realism has been applied to new realms and in the process it has been transformed into sur-realism, magic-realism, super-realism, hyper-realism.

Sometimes these new versions of realism involve a deliberate mixing of external elements (from the world of nature) and internal elements (from the world of the psyche). Linear perspective continues to play a significant role in these explorations. But it is often used in conjunction with other methods. Delvaux frequently uses curved perspective in his streets. Dali moves subtly between regular perspective and anamorphosis. Magritte uses what appears to be linear perspective but then deliberately plays with its underlying principles of occlusion and transparency. Hence walls, which traditionally block out objects are often transparent and windows which are traditionally transparent are often that which block a view in Magritte's paintings. These experiments are spreading to other media. The recent film, *Toys* (1992), adapts Magritte's painting *Golconda* (Menil Foundation, Houston) while playing on the paradox of the window as a mirror.

Implicit in these experiments is an insistence that artists are not bound to a one-to-one correspondence between object and representation. Or to put it positively, artistic freedom is increasingly seen in terms of demonstrating alternatives. Hockney's combinations of photography and painting are a further expression of these trends. In Hockney's case there is also another explicit concern. Linear perspective, he claims, created a wall between the viewer and the object represented. Inverted perspective, according to Hockney, offers a way of removing that wall and integrating both viewer and representation within the same space.[9] Artists such as Rauschenbach have offered yet another reason for inverted perspective: it corresponds, they claim, to visual experience.[10] This idea emerged in the circle of Florenskij in the 1920's and has became accessible to the West partly through the writings of Shegin.[11]

5. Alternative Picture Planes

Since the early nineteenth century there has been a growing fascination with recording visual space as opposed to geometrical space, and this has been an important stimulus for the exploration of alternative picture planes. The most obvious versions entail imitating the convex surface of the eye as in the work of Barre and Flocon that was made accessible through an excellent translation by Robert Hansen,[12] who developed his own method of hyperbolic perspective[13] and subsequently explored the history of the subject. According to Hansen this is not simply a fascination with subjective, psychological factors. The challenge lies in developing objective methods to record visual experience.

A recent exhibition organized by Marcia Clark[14] confirms that this quest is about more than simply recording visual space from a single viewpoint as was traditionally the case. Artists are intent on capturing a series of viewpoints simultaneously. This is one of the incentives not only for Termes' spheres (see the seventh contribution in this section), but

also for his photographs and paintings on polyhedral surfaces. Other artists, Hockney among them, are explicitly searching for means to incorporate the dimension of time into their representations of space. This new interest in dynamic aspects of vision accounts for many of the recent experiments with alternative picture planes using spherical perspective, cylindrical perspective, hyperbolic, fisheye,[15] tetraconic,[16] polyconic and many another variant.

Seventeenth century artists developed a perspectival peep show (*perpektyfkas*) which invited viewers to use a "roving eye" to explore interiors. The work of Frisia (see the eighth contribution in this section), continues this tradition with a curious twist. His work shows us rooms and environments from an interior viewpoint, yet seen outside. Indeed, there is a curious way in which twentieth century experiments can be seen as exterior- or external-izations of spaces which are deliberate plays with the ambiguities of interior-exterior. If linear perspective traditionally captured a particular viewpoint, the new alternative projection methods are integrating a series of viewpoints.

6. Transformations

The incentives for exploring alternative projection methods have not been simply perceptual. The rise of photogrammetry, particularly with the advent of satellite photography, has raised a series of practical challenges. How does one translate a view of the spherical earth from space onto the flat surface of a map? Even very serious cartographical publications have explored the potentially humourous consequences of projecting a human face onto a series of map projections.[17] Meanwhile artists such as Barre and Flocon have studied Renaissance projections (e.g. Postel) and Frisia (cf. the eighth contribution) has incorporated Mercator projections and planisphere projections into his work. The rise of holography has seen new uses of spherical, cylindrical, conical and other curved surfaces both for recording and viewing of these images.

7. Ambiguities and Illusions

The nineteenth century drew attention to a series of geometrical optical illusions. The twentieth century has continued to be fascinated by these and there have been some attempts to explain them in terms of perspectival experience. Some artists have recognized that these illusions lend themselves to artistic treatment. Reutersvärd (1934) was among the first to do so with the impossible figure that came to be known as the Penrose tribar (1958, cf. elsewhere in this book), and as Ernst[18] has shown, inspired a whole series of visual commentaries including Escher's famous *Waterfall* (1961). Indeed a number of Escher's works are deliberate juxtapositions of sections in linear perspective which, when combined, result in impossible spaces. This is yet another expression of explorations of non-correspondence between object and representation.

8. Virtual Reality

In the Renaissance, artists used instruments such as the mirror and the perspectival window as a means of recording the visible world with linear perspective. In the latter twentieth century, images of the mirror and the looking glass continue to be used, now in the context of the computer screen, and with a new goal of seeing the hitherto invisible. As Ivan Sutherland put it: "We lack corresponding familiarity with the forces on charged particles, forces in non-uniform fields, the effects of non-projective geometric transformations, and high energy, low friction motion. A display connected to a digital computer gives us a chance to gain familiarity with concepts not realizable in the physical world. It is a looking glass into a mathematical wonderland."[19] This visionary statement in 1965 was one of the starting points for Sutherland's head mounted display

(first published in 1968), experiments at N.A.S.A and led to the emerging fields of visualisation and virtual reality. Renaissance perspective sought to look through the window: virtual reality is attempting a new type of immersion "through the looking glass".[20]

Virtual reality raises new problems of perspective. Even very simple systems such as the PC based Superscape Virtual Realities System entails a basic perspectival space for the general environment within which there are other viewpoints that can be manipulated simultaneously: from either inside or outside an automobile, from behind a person or off to the side of a helicopter. Renaissance perspective typically involved either interiors or exteriors. Virtual reality is introducing new, dynamic interplays between egocentric and exocentric viewpoints, including the ability to move through both walls and windows enabling seamless transitions between interiors and exteriors. In the emerging field of cyberspace these principles are being extended as navigational tools among data cells in visual databases.[21] In the eighteenth century Kant described verbally how we use perspectival space as a tool for orientation in our mind.[22] Virtual reality and cyberspace are helping us to visualize this process. Spatial, perspectival co-ordinates are basic, not only in the virtual cockpit, but in all our activities.

9. Scale and Fractals

As mentioned earlier, one of the fundamental tenets of Renaissance perspective was the inverse size-distance law which stated that if one doubled the distance the represented size was one half, if one trebled the distance the size was one third and so on. Mandelbrot's article about the size of the coast of Britain[23] implicitly introduced a spanner into this assumption by showing that size was a function of scale as well as distance. In a sense we have been vaguely aware of this ever since the seventeenth century. The shape of an ordinary image is transformed entirely when we change the scale of its image radically in in a telescope or a microscope. Fractals may have brought this problem into focus, but fractals assume a principle of iteration, i.e. their basic patterns are repeated and hence remain independent of scale. Hence they do not solve the problem which they have raised. What is needed is a new approach to perspective that takes into account scale as well as distance, whrereby any given shape only applies within a given range of scales. This is increasingly important in a world where we travel between scales with greater frequency.

10. Conclusions

If perspective is defined in a narrow sense as linear perspective then one of the major reasons for its continued popularity is a growing historical awareness which seeks both to understand methods developed in the Renaissance and apply new technologies in the analysis thereof. Some of the major themes of the earlier treatises such as regular solids remain significant to this day.

Yet there are significant contrasts between Renaissance methods and modern developments. The Renaissance paid lip service to equations between perspective and vision, while at the same time linking perspective increasingly with geometry and commiting themselves to recording geometrical space of the physical world. Some twentieth century artists have continued this tradition in their explorations of realism, hyper-realism, and sur-realism. Others have abandoned this commitment and focussed increasingly on the exploration of visual space, both exterior and interior. This has led to new goals of art in terms of exploring perceptual, mental, dream, psychological and even psycho-pathological states. As a result, whereas Renaissance artists focussed attention on linear picture planes, twentieth century artists are exploring many alternative shapes of

picture planes. They are also contradicting the traditional transparency-occlusion principles of perspective in their quest for artistic freedom. Hence whereas Renaissance artists established a one to one correspondence between object and representation, twentieth century artists strive to demonstrate the contrary.

The rapid development of computer graphics, which allows one to transform one kind of picture plane into another simply by altering the algorithms for the perspectival grids has added new vigour to these experiments. So too has the continued study of psychological aspects of spatial representation and perception. Optical illusions and visual ambiguities have instilled a new playfulness into these explorations of space as has the development of virtual reality. The rise of fractals have made us aware that scale is a factor that needs to be taken into account. Perspective in this sense has yet to be developed even though linear perspective is now some five hundred and seventy years old. Whether in the old or the new sense perspective remains one of the most fascinating expressions of links between mathematics and art.

Notes

[1] H. H. Arnason, *History of modern art*, London: Thames and Hudson, (1969), 1983, p. 9

[2] This is being prepared by a team under the direction of Professors Dalai Emiliani, Grayson and Maccagni

[3] Cited in note 10 of the article by Geatti and Fortunati below.

[4] See the remarkable English edition by A. I Sabra, *The Optics of Ibn Al-Haytham*, London: Warburg Institute, 1989, vol. 1-2. Two further volumes are expected.

[5] a discussion of these theories see the author's "Panofsky's perspective : a half century later", in *La prospettiva rinascimentale*, ed. Marisa Dalai Emiliani, Florence: Centro Di, 1980, 565-584.

[6] Abraham Bosse, *La maniere universelle de Monsieur Desargues*, Paris, 1665, p.58 : "Pour prouver qu'il ne faut pas dessiner n'y peindre comme l'oeil voit".

[7] See, for instance, Sir Ernst Gombrich, "Review lecture: mirror and map", *Philosophical transactions of the Royal Society, B. Biological Sciences*, London, vol. 270, (n. 903), 1975, p. 119-149,

[8] See, for instance, J. J. Gibson, *Ecological optics*, 1979. For a discussion of how Gibson changed his terms several times in the course of his writings see the author's *Literature on perspective*, (as in note 2).

[9] See, for instance, David Hockney, "Perspective", in *An Art Design Profile. David Hockney*, London: St. Martin's Press, 1988, p. 84-89.

[10] This was published in *Leonardo*, vol. 16, 1983 and is listed in the bibliography to this book.

[11] See, for instance, L. F. Shegin, *Die Sprache des Bildes*, Dresden: VEB Verlag der Kunst, 1980

[12] Barre and Flocon, *Curvilinear perspective*, Trans. and ed. Robert Hansen, Berkeley: University of California Press, 1987.

[13] See the bibliography at the end of the book.

[14] Marcia Clark, ed., *The world is round. Contemporary panoramas*, New York: the Hudson River Museum, 1987.

[15] Michael Moose, "Guidelines for constructing a fisheye perspective", *Leonardo*, vol. 19, no. 1, 1986, pp.61-64.

[16] Kenneth R. Adams, "Tetraconic perspective for a complete sphere of vision", *Leonardo*, vol. 9, no. 4, 1976, pp. 289-291. A more thorough discussion of these alternatives is found in chapter three of the author's forthcoming *Literature of perspective*.
[17] See, for instance, W.R. Tobler, *An experiment in the computer generalization of maps*, Ann Arbor: Office of research administration, 1964, 2 vol.
[18] Bruno Ernst, *Abenteuer mit unmöglichen Figuren*, Berlin:Taco, 1987.
[19] Ivan Sutherland, "The Ultimate Display", *Proceedings of the IFIP Congress*, 1965, pp. 505-508.
[20] Cf. Ken Pimentel and Kevin Teixeira, *Virtual reality. Through the looking glass*, New York: Windcrest/Mcgraw Hill, 1993.
[21] See, for instance, plates 1-3 in *Cyberspace: First Steps*, ed. Michael Benedikt, Cambridge, Mass.: M.I.T. Press, 1991.
[22] Immanuel Kant, "Was heisst sich im Denken orientieren?", *Berliner Monatsschrift*, Oktober, 1786. For a discussion in English see Ernst Cassirer, *The philosophy of symbolic forms*, New haven: Yale University Press, 1955, vol. 2, p. 93.
[23] B. Mandelbrot, "How long is the coast of Britain? Statistical self-similarity and fractional dimension", *Science*, vol. 155, p. 636-638.